

# Procedural Function-based Spatial Microstructures

*<http://hm.softalliance.net/>*

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# Outline

- ◆ Heterogeneous objects modelling
- ◆ Natural and artificial microstructures
- ◆ Problems with surfaces & voxels
- ◆ Using real functions
- ◆ Regular and non-regular procedural microstructures
- ◆ Direct rendering and fabrication

# Function Representation FRep

- ◆ Uniform representation of multidimensional point sets as

$$F(X) \geq 0$$

- ◆ Function  $F(X)$  evaluation procedure traversing the construction tree structure
- ◆ Leaves: primitives
- ◆ Nodes: operations + relations

# Constructive Hypervolume Model

Hypervolume is a multidimensional point set with multiple attributes

$$o = ( F(X), S_1(X), \dots, S_k(X) )$$

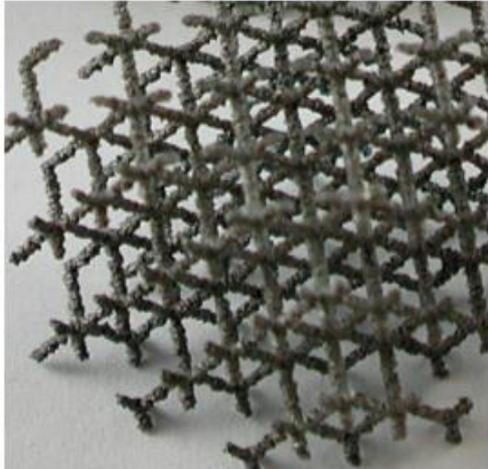
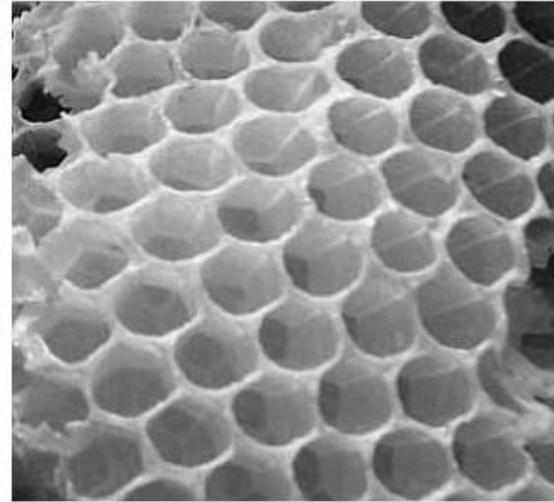
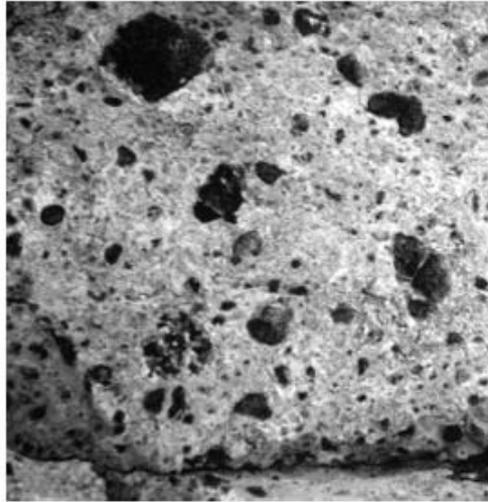
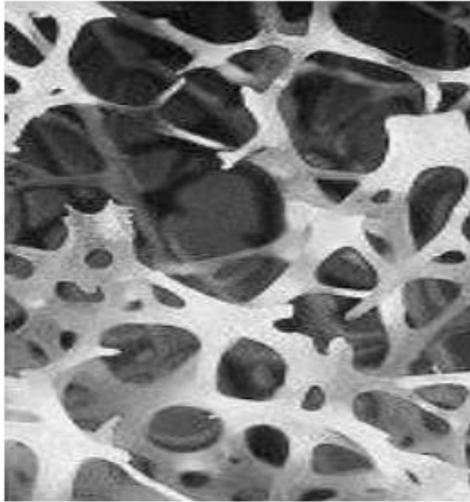
$F(X)$  – FRep of geometry

$S_i(X)$  – attributes based on FRep  
space partitions

# Heterogeneous objects

- ◆ Internal structure with non-uniform volumetric distribution of properties (density, color, transparency, etc.)
- ◆ Entities of different dimensionalities
- ◆ Gradually varying material distribution in CAD/CAM and fabrication
- ◆ Physical simulations, geological and medical modeling and rendering

# Natural and artificial microstructures



# Problems with surfaces & voxels

- ◆ **Size and processing time**
  - 100s Mb polygons,  $>10^{10}$  voxels
- ◆ **Validity and precision**
  - cracks and approximations
- ◆ **Parameterization and operability**
  - blends, offsets, deformations
- ◆ **Manufacturability**
  - STL problems are amplified by the geometric complexity of microstructures

# Procedural microstructures

Procedural generation of the defining function  $F(X)$  value at the given point such that geometry of the entire microstructure is described as

$$F(X) \geq 0$$

Constructive model based on R-functions:

$$f_3 = f_1 \vee_{\alpha} f_2 \quad \text{for the union;}$$

$$f_3 = f_1 \wedge_{\alpha} f_2 \quad \text{for the intersection;}$$

# Regular infinite lattices

Periodic infinite slabs

$$s_x(x, y, z) = \sin(q_x x + p_x) - l_x$$

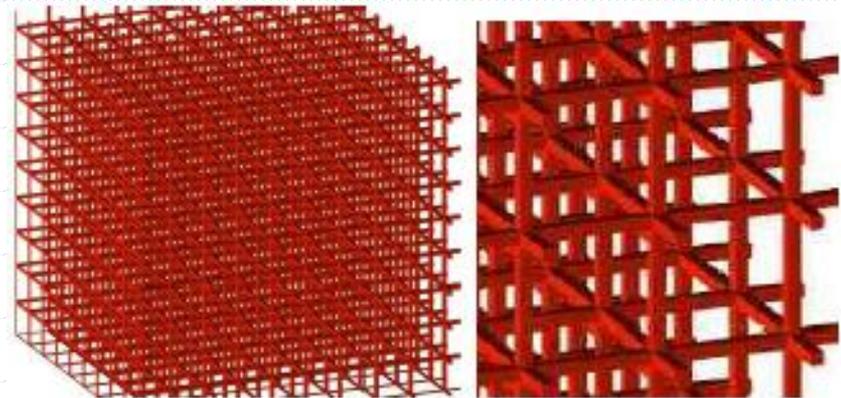
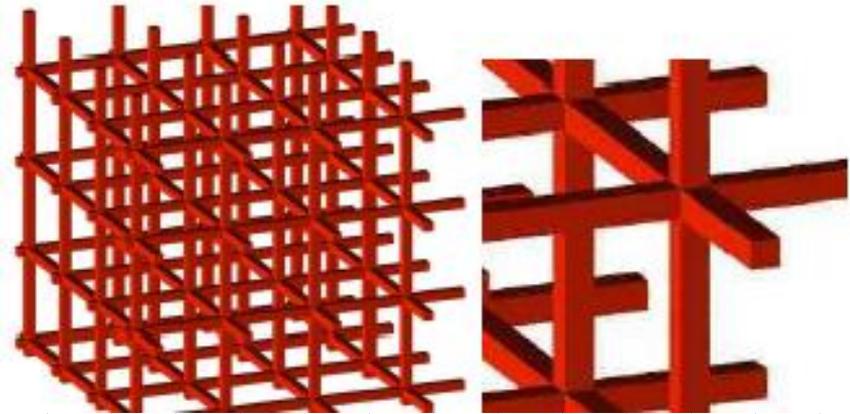
Rods: intersection of slabs

$$r_x(x, y, z) = s_y \wedge_\alpha s_z$$

Lattice: union of rods

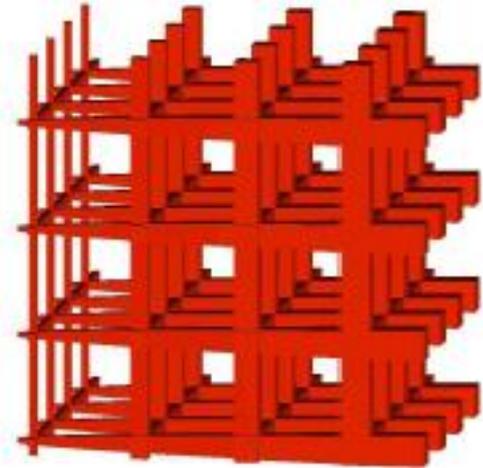
$$g(x, y, z) = r_x \vee_\alpha r_y \vee_\alpha r_z$$

$$g(x, y, z) = (s_y \wedge_\alpha s_z) \vee_\alpha (s_x \wedge_\alpha s_z) \vee_\alpha (s_x \wedge_\alpha s_y)$$

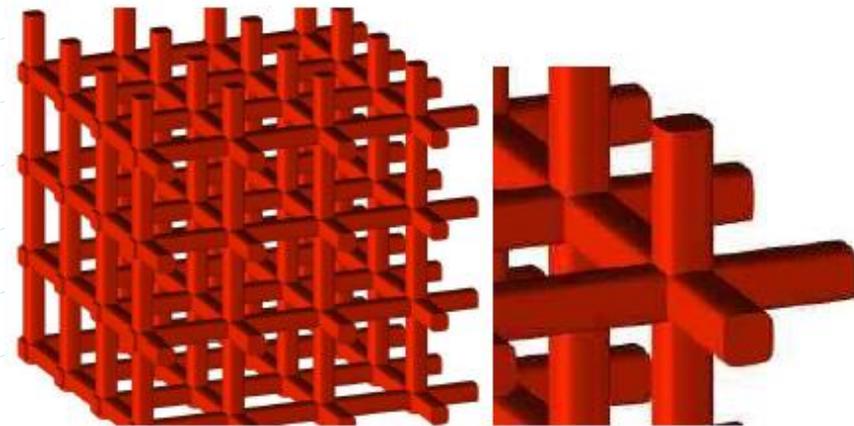


# Variations of lattices

- ◆ Variable rod thickness

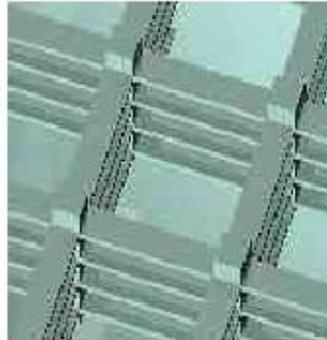


- ◆ Smoothed rods

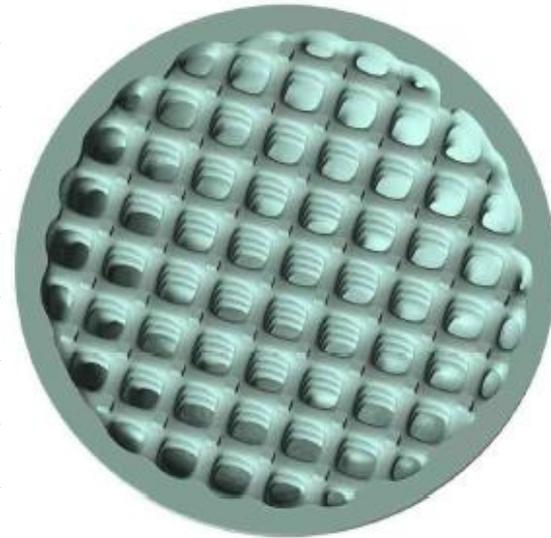


$$f_1 \wedge_b f_2 = (f_1 \wedge_\alpha f_2) + \frac{a_0}{1 + \left(\frac{f_1}{a_1}\right)^2 + \left(\frac{f_2}{a_2}\right)^2}$$

# Combining with a shell



Truncation of a lattice by a solid and union with its shell



Blending union between rods and with a shell

# Parameterization by distance

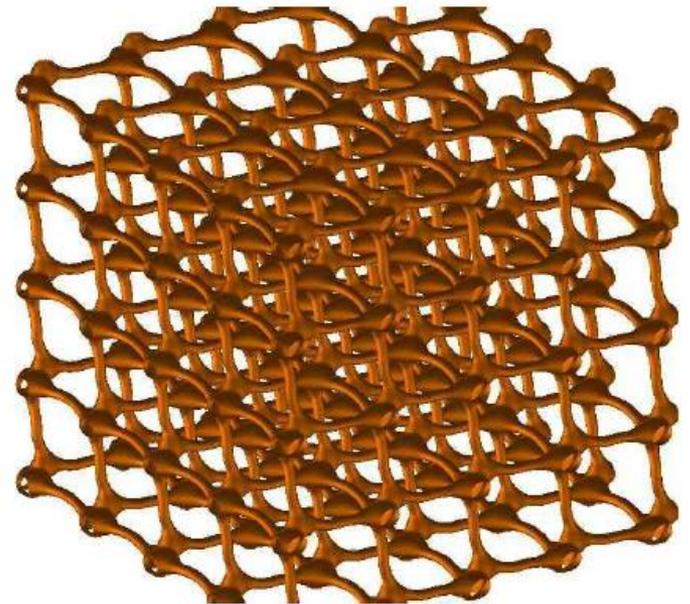
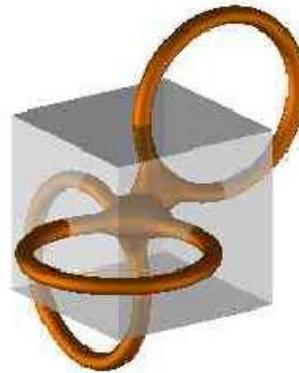


Lattice step decreases closer to the surface

# Cellular microstructures

Replication of a unit cell with periodic space mapping:

$$x' = \text{perodic}(x)$$



# Cellular microstructures

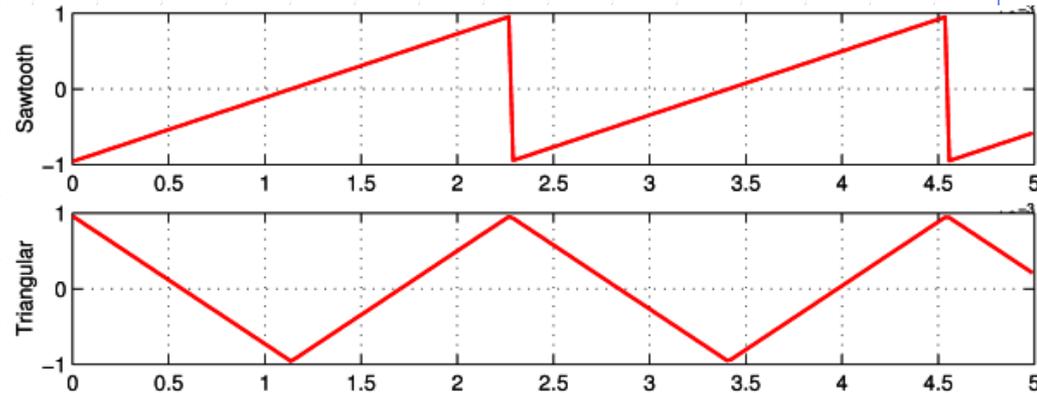
Non-symmetric cell -

sawtooth

Symmetric cell -

triangular

Sawtooth



Triangular

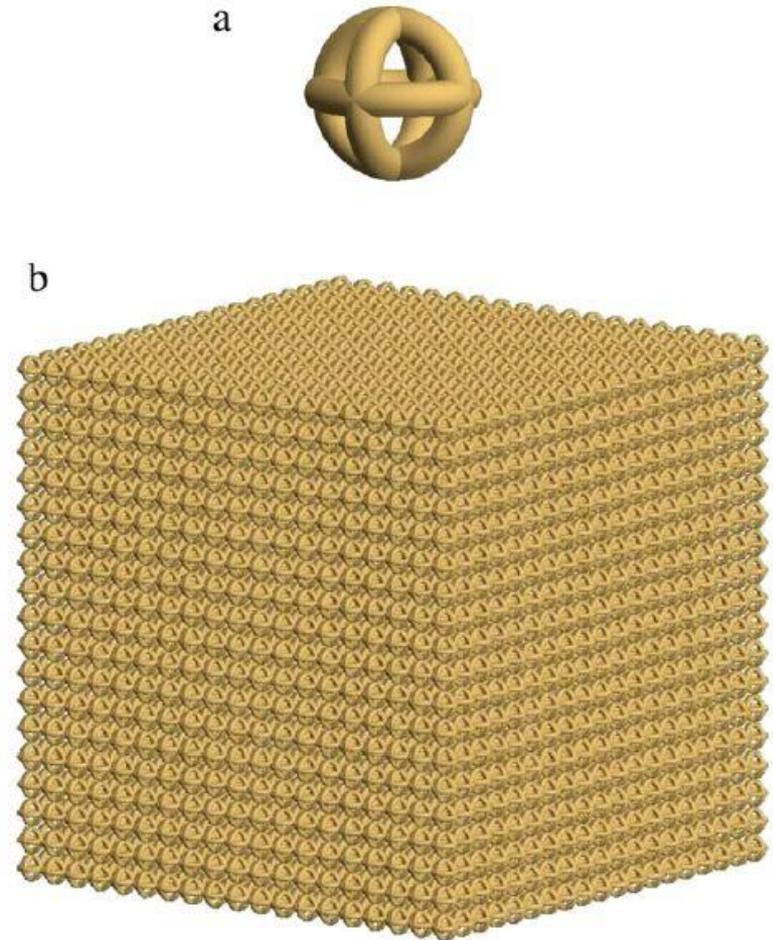
$$g(t) = \frac{1}{2} + \left( \frac{t}{a} - \text{floor} \left( \frac{t}{a} + \frac{1}{2} \right) \right)$$

$$g(t) = \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \left[ \sin \left( \pi \frac{t}{a} \right) \right]$$

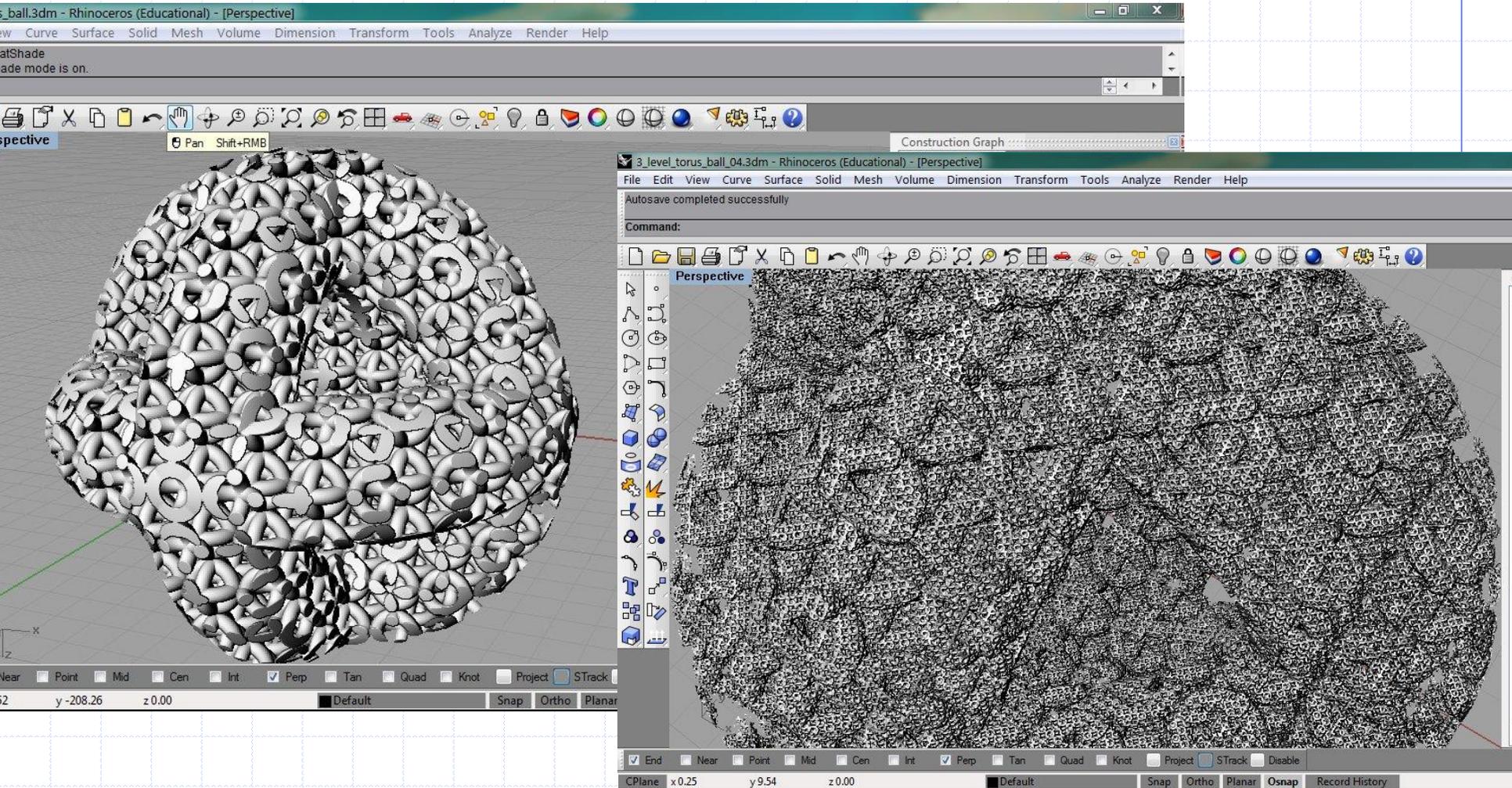
where  $a$  is a period of the function.

# Cellular microstructures

Replication of a unit cell (a: union of three tori) with periodic space mapping (b: sawtooth or triangle)



# Multi-scale nested FRep structures



3 levels of nested tori balls

# Metamorphosis

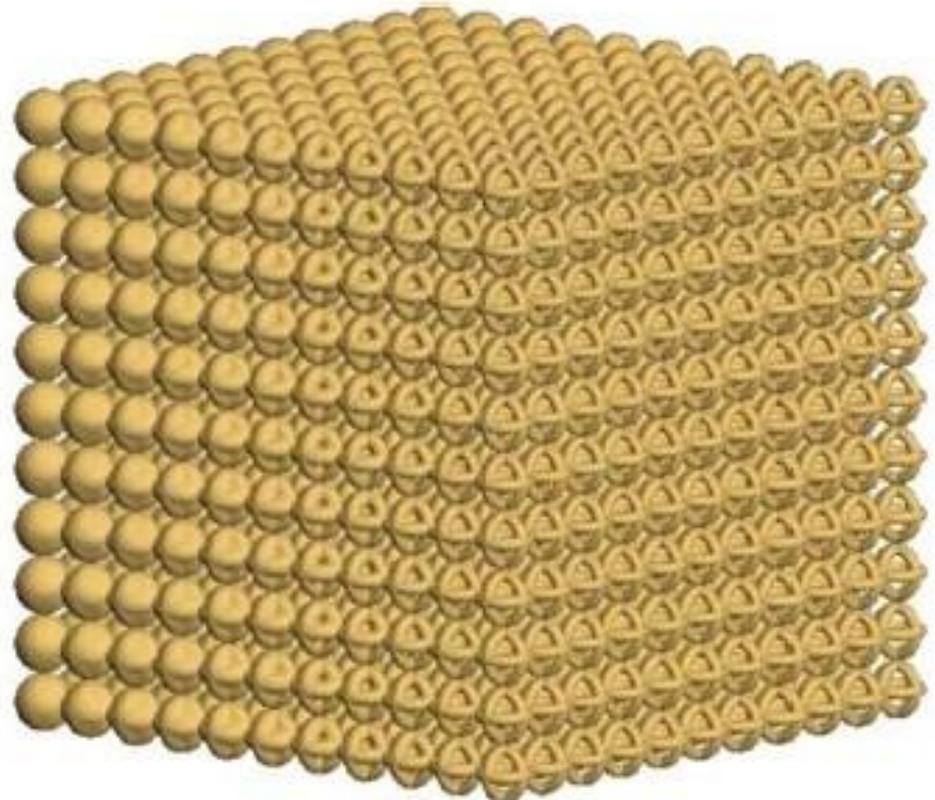
Spatial metamorphosis from a ball to a union of three tori:

$$f = f_1(1-t) + f_2t$$

$$t \in [0,1]$$

*f<sub>1</sub> is a ball*

*f<sub>2</sub> is a union of three tori*



# Spatial interpolation

Cell metamorphosis:

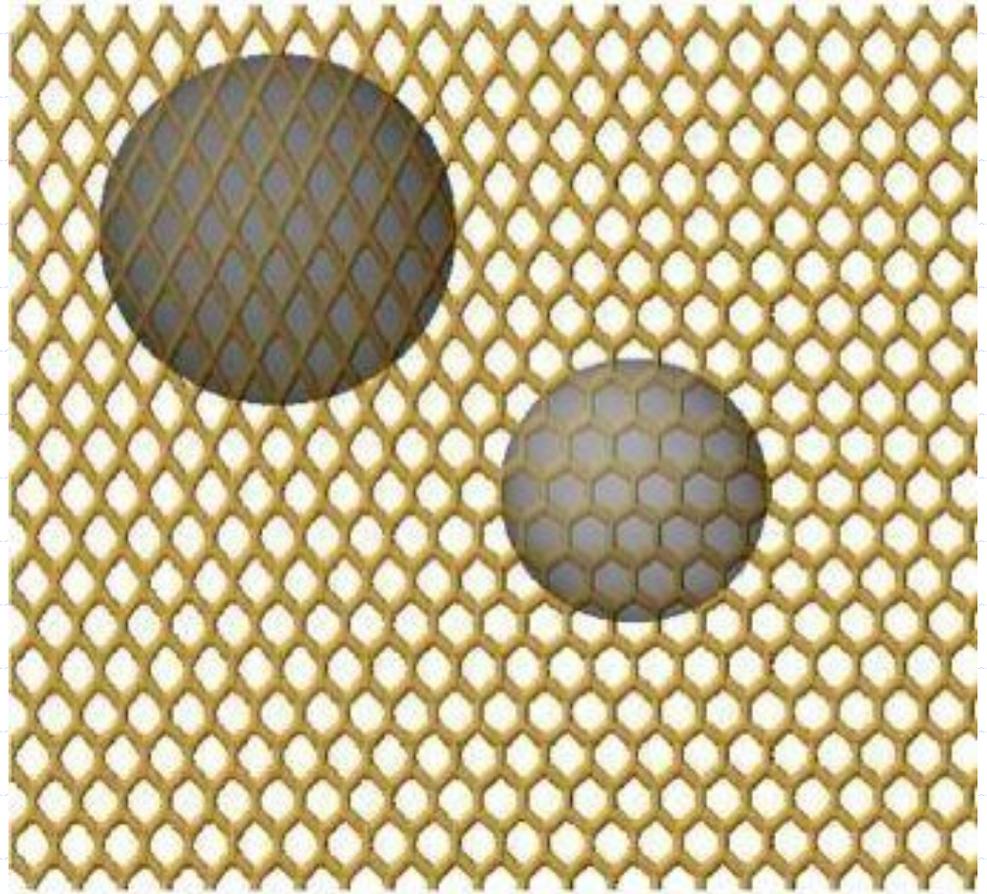
$$f = f_1(1-t) + f_2t$$

$$t \in [0,1]$$

$f_1$  is a rhomboid

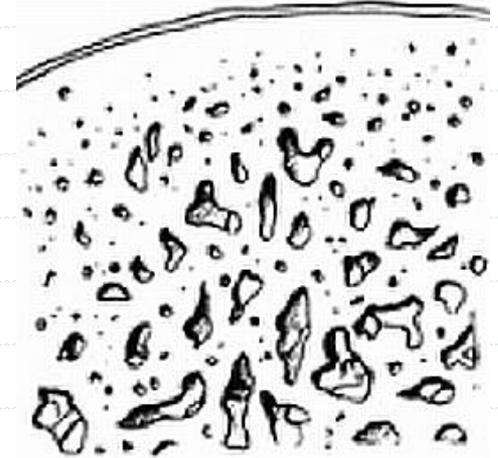
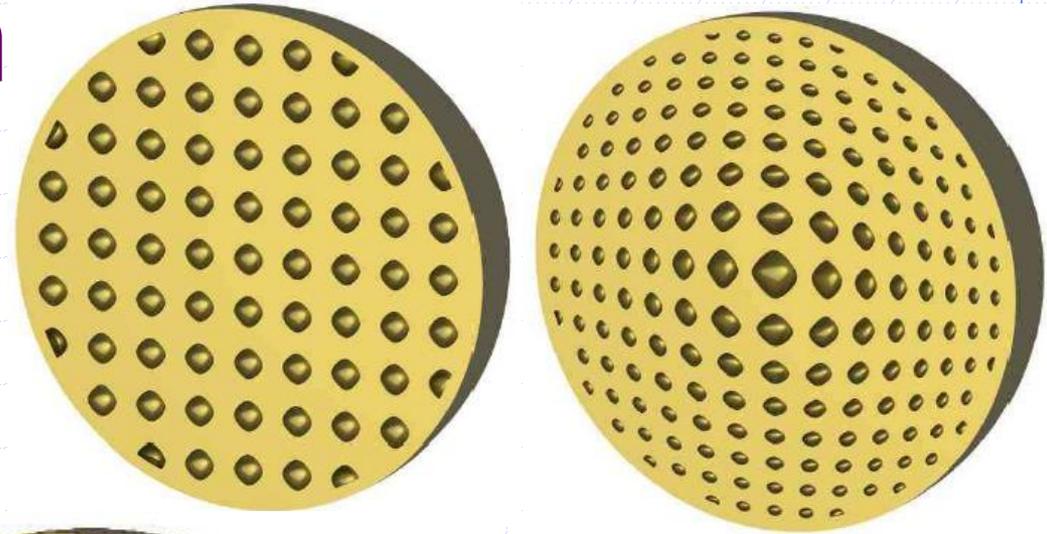
$f_2$  is a hexagonal lattice

*or transfinite interpolation*



# Porous media

1. Basic pore replication
2. Distance dependency
3. Adding noise



# Direct rendering

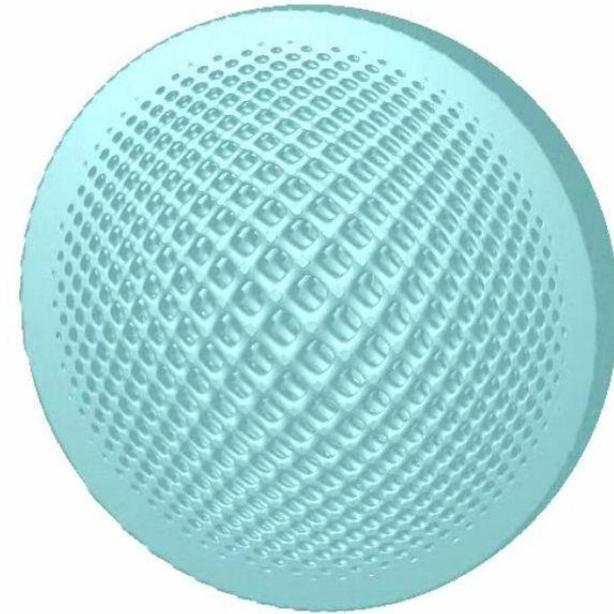
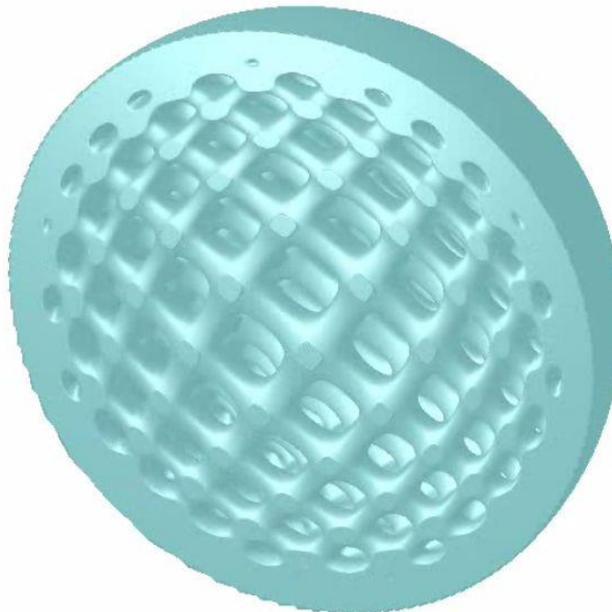
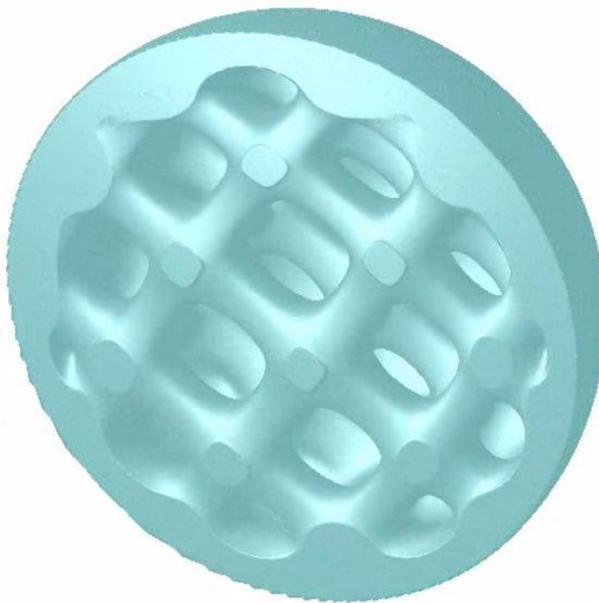
Real-time ray-tracing on GPU

independent of the microstructure density

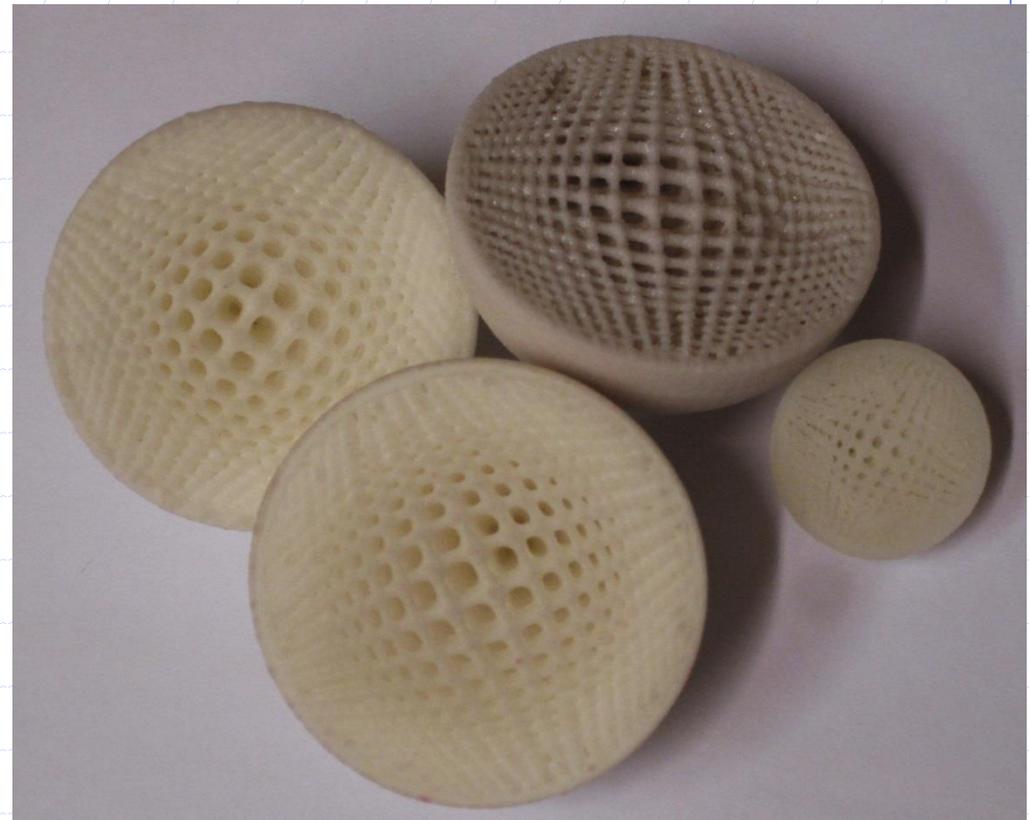


# Direct rendering

with blending to the shell and between rods



# Towards direct fabrication



- ◆ 3D Systems Sinterstation
- ◆ ZCorp 3D printer
- ◆ Stratasys Dimension
  
- ◆ STL problems
- ◆ Proprietary protocols

3D prints by the Centro de Tecnologia da Informao, CTI, Brasil

# Applications

- ◆ Additive manufacturing / 3D printing
- ◆ Mechanical and bio-engineering
- ◆ Geological exploration
- ◆ Medicine
- ◆ Digital art and entertainment
- ◆ Creativity for disabled children - SHIVA project

# Applications in 3D Printing



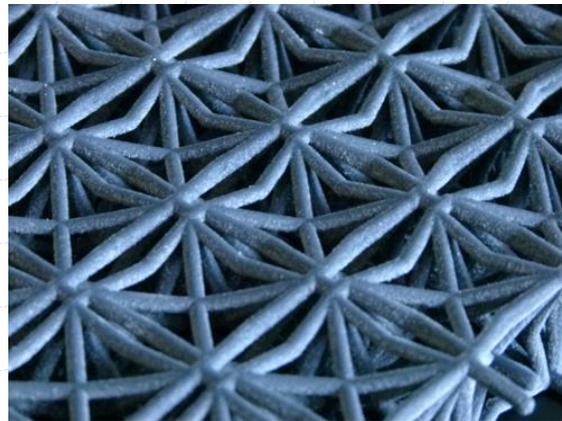
Chess pieces (resin & stereolithography)



Solver pendant (3D printed wax, casting)



Pavilion (concrete 3D print, D-shape)



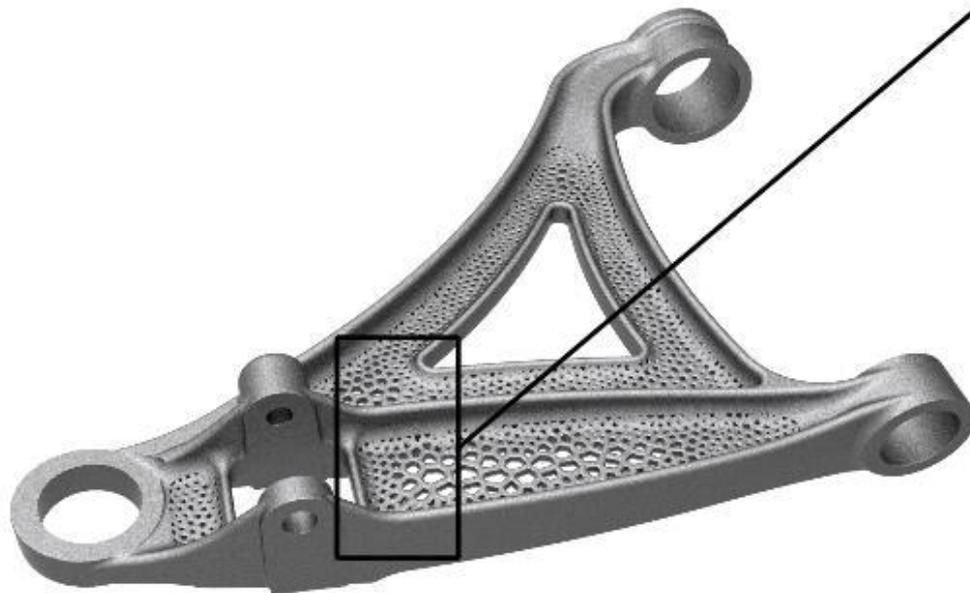
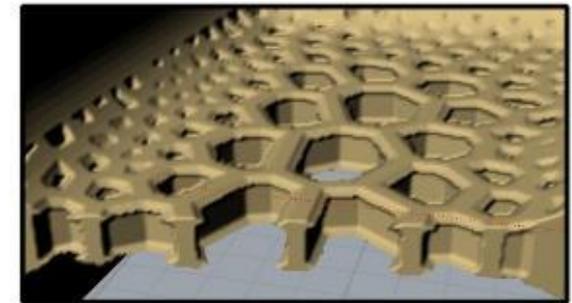
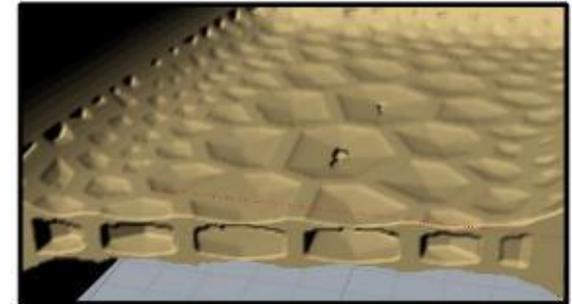
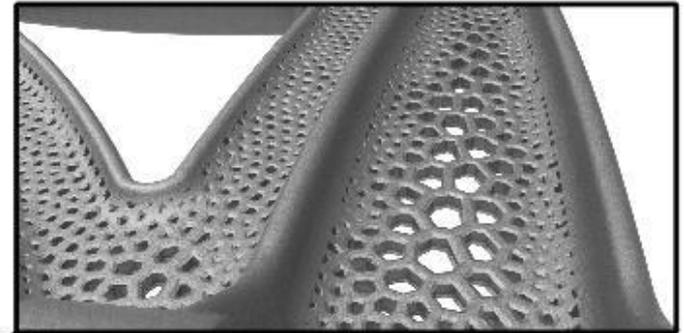
Construction pattern (plastic, ZCorp)



Bonbonniere (wood, milling machine)

Adaptive variable cellular and internal structures based on direct FEA and simulation feedback linked to dynamic generative parameters.

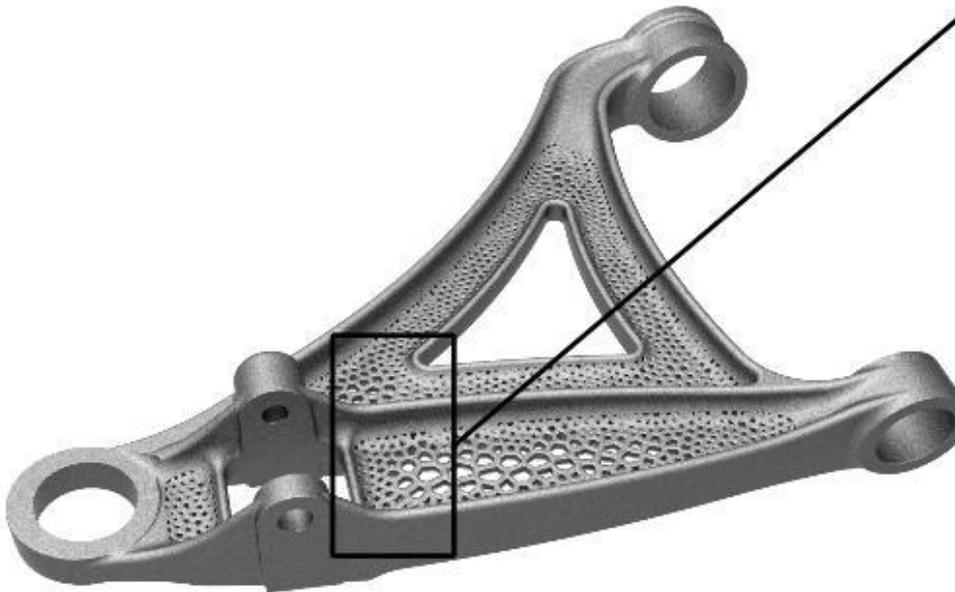
Ex 1: Simulation optimized & parameterized cellular structure with porous skin.



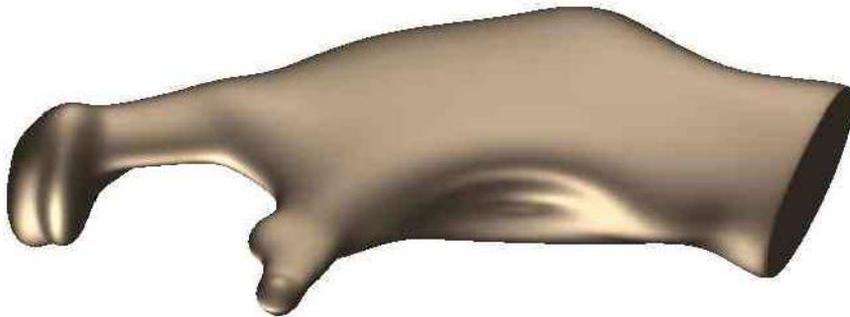
## Next Steps – Advanced Designs w/ Automatic Closed Loop Simulation Feedback

Adaptive variable cellular and internal structures based on direct FEA and simulation feedback linked to dynamic generative parameters.

Ex 1: Simulation optimized & parameterized cellular structure with porous skin.



# Lattice scaffold for a jaw bone



Initial jaw bone model



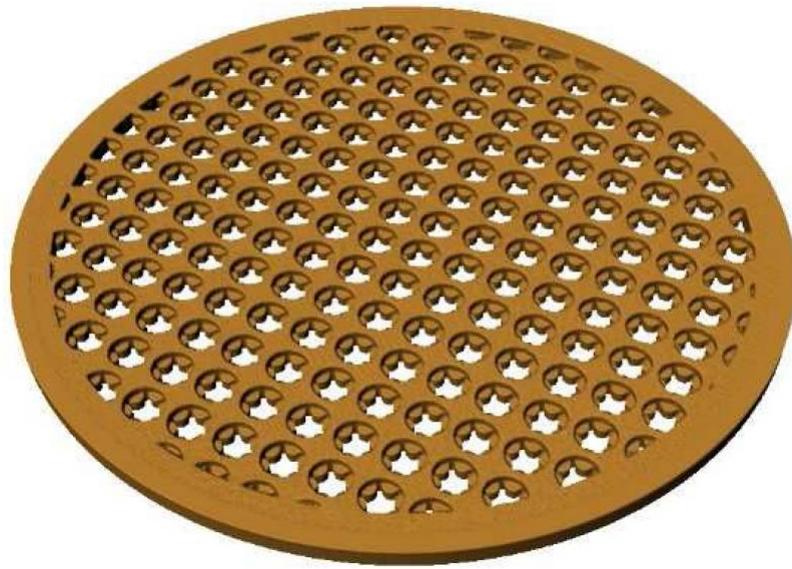
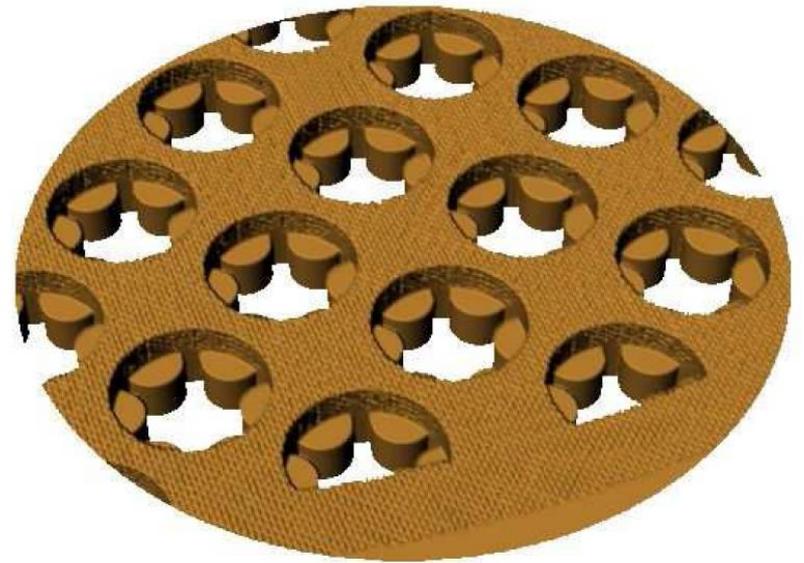
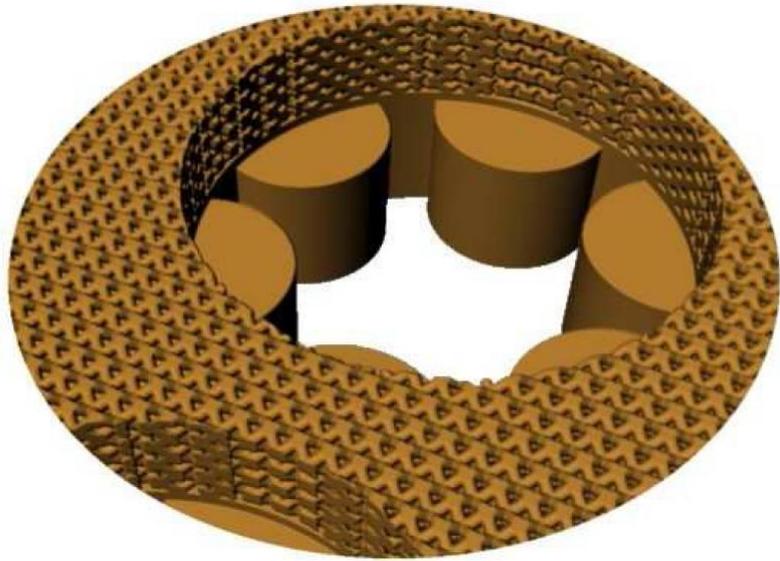
Union of lattice with bone shell



Lattice truncated by bone

Jaw bone model by  
Denis Kravtsov

# Filter design

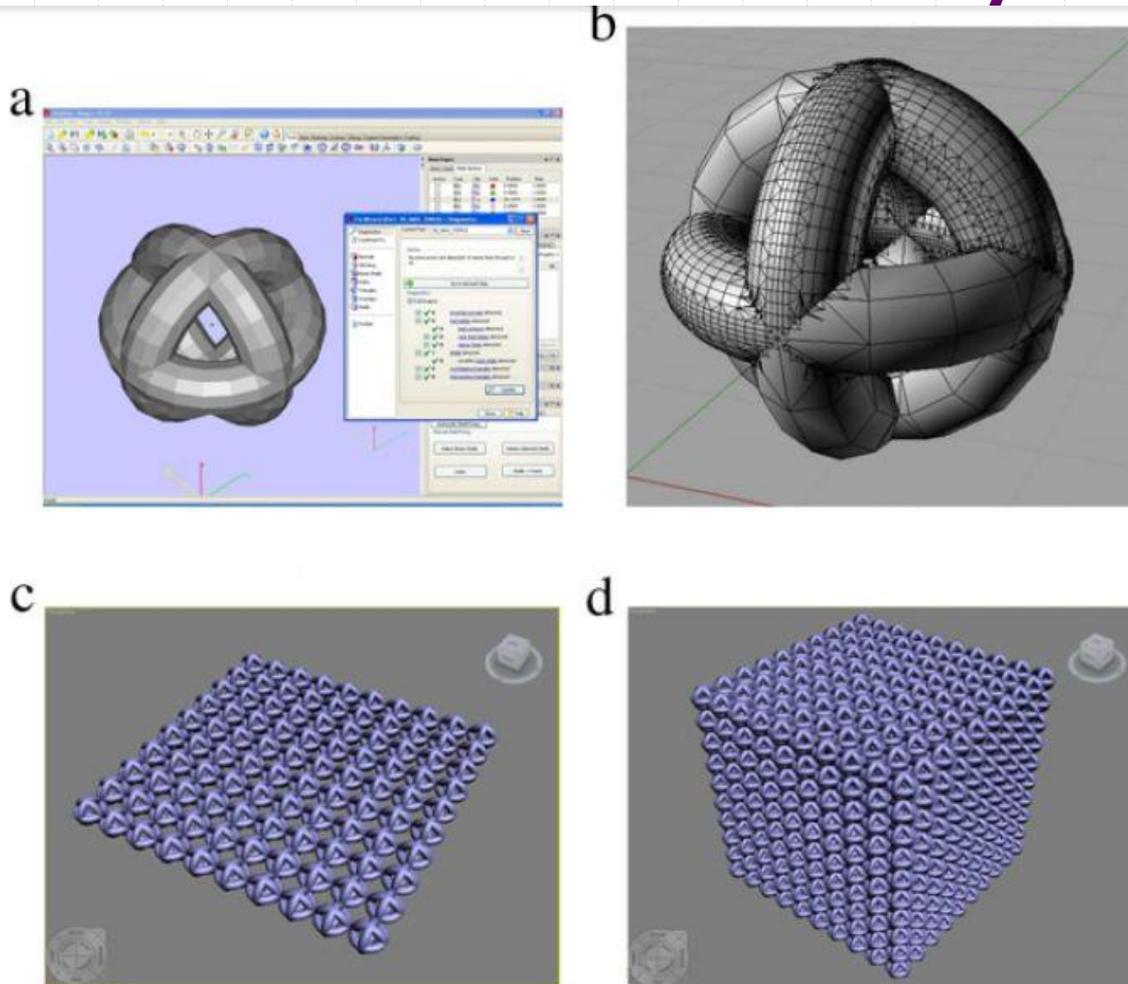


# Surface design



Set-theoretic difference between the surface and the volume microstructure

# Experiment with CAD systems



All CAD systems have failed the array of 100 tori balls

# Conclusions

Polygon-free and voxel-free approach to

- ❖ Interactive modeling
- ❖ Real-time rendering
- ❖ Fabrication (ongoing)
- ❖ Fitting and analysis (future work)

Step towards procedural multiresolution modeling on micro- and nano-levels with infinite “zoom”.